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ON THE VALIDITY OF THE CARMAN-KOZENY EQUATION IN RANDOM FIBROUS MEDIA

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Abstract. The transverse permeability for creeping flow through unidirectional random arrays of fibers/cylinders has been studied numerically using the finite element method (FEM). A modified Carman-Kozeny (CK) relation is presented which takes into account the tortuosity (flow path) and the lubrication effect of the narrow channels. The proposed relation is valid in a wide range of porosities compared to the classical CK equation. The proposed general relationship for the permeability can be utilized for composite manufacturing and also for validation of advanced coarse models for particle-fluid interactions.

1 INTRODUCTION

Fibrous materials are widely used in industry as well as our daily life due to their flexibility, strength, and most importantly, permeability [1]. One of the common examples of such materials is unidirectional fibre reinforced composites. These composites consist of a large number of 2D uni-directional fibres/cylinders embedded in the matrix phase at random locations perpendicular to the transverse plane. The goal is to determine the overall behaviour (i.e. mechanical and/or transport properties) of such a material provided that material properties and geometry of the matrix and fibres are known.

In most of the composite manufacturing processes (e.g. resin transfer moulding (RTM), autoclave process, vacuum infusion, etc.), the permeability is critically important. During the last decades, several researchers tried to develop a simple model that could predict the permeability of the fibrous medium as a function of solid volume fraction by incorporating computational/numerical or experimental measurements, see for example [1-5] and references therein.

In this respect, two distinct approaches have emerged; (i) the lubrication approach for dense systems and (ii) the cell method valid at larger porosity (dilute regime). Based on the lubrication theory at high volume fraction, the pores of a porous medium can be considered as a bunch of capillary tubes which are tortuous or interconnected in a network. An expression for the permeability as a function of solid spacing, for regular fibre arrays, was derived which closely matches the numerical results at high volume fractions [6-7].

On the other hand, for dilute systems, i.e. at high porosities, the cylinders are widely spaced, and the cell method is appropriate. It assumes that the cylinders are spaced far away so that the region can be divided into practically independent cells. Thus, the arrangement and shape of the fibres has no effect on the permeability. In fact, at this limit, the permeability can be obtained by adding the resistance of individual particles/fibres. The dependence of permeability in this limit involves logarithmic, linear and quadratic functions of the solid concentration [8-10].

The earliest and most widely applied models in the composites literature, in intermediate porosity regimes, for predicting permeability are capillary models such as the Carman-Kozeny (CK) equation for random sphere packings [11]. While some studies have reported success with this relation, discrepancies are also reported. Gutowski et al. [12] found that the Carman-Kozeny equation could give a good fit to the axial permeability of unidirectional reinforcements, while there was a certain deficiency for the transverse permeability. It turns out that for the permeability prediction, the geometrical arrangement of fibres must be taken into account [13]. Although most parameters in this model can be calculated based on the geometrical structure of fibre reinforcements, it also has an empirical constant (the CK coefficient) determined by experimental data that can not be used directly for other types of fibre reinforcements.

In a recent work [14] the trend from low to high porosity extremes for regular structures is described, however, there is a lack of correspondence in the intermediate range of porosity for disordered fibre arrays. It is mainly due to difficulties in analyzing the precise geometry and topology of the pore system and proper characterization of the stochastic nature of the pore structure.

Our objective in this paper is to present a finite element based model that allows utilizing real 2D/3D microstructures in calculating the permeability of the fibrous materials. The macro description of the fluid flow equations and the numerical tools employed to solve these equations, are presented in the next Section. The permeability values obtained from our numerical results are compared with previous theoretical and numerical data for random and ordered configurations in Section 3. We propose a modification to the original CK equation as an attempt to combine our various simulations in a wide range of porosity and relate macroscopic permeability to the micro properties and geometry of fibrous materials. The paper is concluded in Section 4 with a summary and outlook for future work.

2 MATHEMATICAL FORMULATION

In this Section we summarize the relations describing the macroscopic flow in fibrous structures and the finite element method (FEM) employed to solve these equations.

2.1 Macroscopic description of the flow equations

The flow description in the porous media is based on the assumption that a Newtonian and incompressible fluid flows under steady-state conditions. The Navier-Stokes (NS), i.e. the conservation of momentum and the continuity equations, i.e. mass conservation, for this case reduce to

$$\begin{aligned} &\nabla.\vec{u} = 0, \\ &\rho\left(\vec{u}.\nabla\vec{u}\right) = -\nabla p + \mu\nabla^2\vec{u}. \end{aligned} \tag{1}$$

where $\vec{u}$, ρ, μ and p are velocity, density of the fluid, viscosity and pressure of the fluid, respectively. When ρ and μ are picked for a given fluid, solution of the flow problem yields the velocity profile for a given pressure gradient (∇p) which is applied as a boundary condition. This information is used to calculate the superficial velocity $\vec{U}$, through the system as

$$\vec{U} = \frac{1}{V}\int_{V_f} \vec{u}dv = \varepsilon\left\langle \vec{u}\right\rangle \tag{2}$$

where $\left\langle \vec{u}\right\rangle$, V, V_f and $\varepsilon = V_f/V$ are the averaged velocity, total volume, volume of the fluid and porosity, respectively. According to Darcy's law for unidirectional flow through a porous medium in creeping flow regime, the superficial fluid velocity is proportional to the pressure drop per unit length ($\Delta p/L$). The proportionality constant being the permeability K, of the medium

$$\vec{U} = -\frac{K}{\mu}\nabla p \tag{3}$$

which strongly depends on the microstructure (e.g. fiber arrangement, void connectivity and inhomogeneity of the medium) and also on porosity. The effect of several microstructural parameters such as fiber shape, orientation, etc. in both creeping and inertial flow for regular structures have been studied in Refs [14-16] and references therein. In the next Subsection the permeability results obtained from FEM simulations for random fiber configurations will be presented.

2.2 Computational method

The FEM software ANSYS® is employed to calculate the superficial velocity and, using Eq. (3), the permeability of the fibrous material. Several issues like the required system size (or number of fibers), random packing generation algorithms, isotropy and heterogeneity of the structure were addressed in [17]. For all of our simulations presented here, we use the Monte Carlo (MC) procedure to generate random fiber structures. All simulated domains contain 800, non-overlapping fibers with minimum inter fiber distance $\delta_{\min}=0.05d$, where d is the diameter of the fibers.

Fig. 1 shows a schematic of a 3D and 2D representation of 200 randomly distributed fibers normal (y) to the flow direction (x) at porosity $\varepsilon=0.6$ with minimum inter fiber distance $\delta_{\min}=0.05d$. Similar to Chen and Papathanasiou [3], a minimum distance is needed to avoid complete blockage. At the left and right boundary pressure is prescribed, at the top and bottom wall (z direction) surfaces and at the surface of the particles/fibers no-slip boundary conditions are applied. Fibers are assumed to be very long so that a 2D solution is representative. A typical unstructured, fine, triangular FEM mesh is also shown in Fig. 1. The mesh size effect is examined by comparing the simulation results for different resolutions (data not shown here). The range of number of elements is varying from 5×10^5 to 10^6 depending on the porosity regime. The lower the porosity the more elements are needed in order to resolve the flow between close, neighboring fibers. To obtain good statistical accuracy, the permeability values were averaged over 10 realizations. Some more technical details are given in Refs [14, 17].

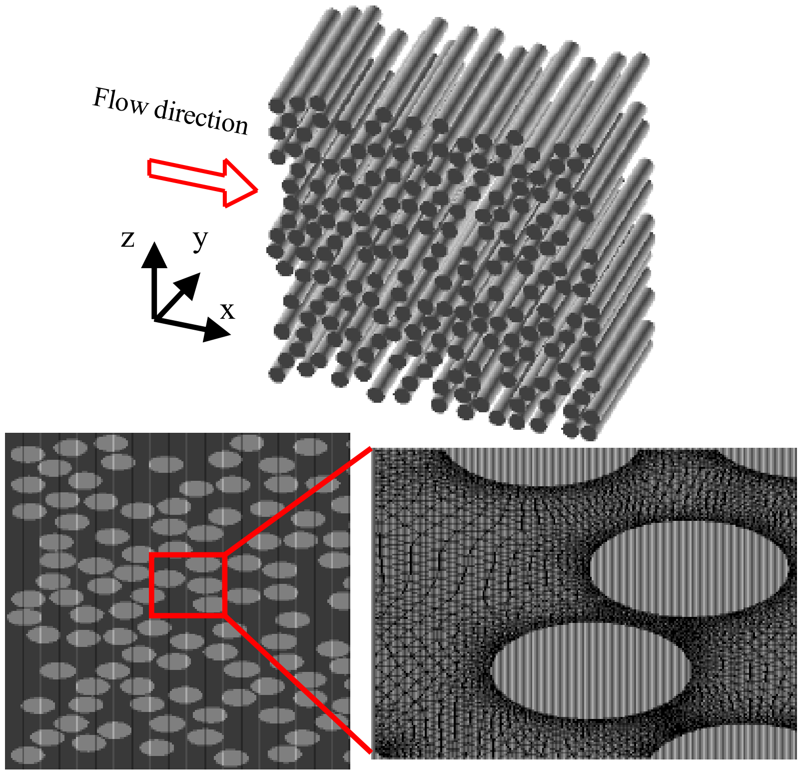

Figure 1: Fiber distributions generated by a Monte Carlo (MC) procedure, with 200 uni-directional cylinders, normal (y) to the flow direction (x), with minimum inter fiber distance $\delta_{\min}$=0.05d (d is the diameter of the fibers) at porosity ε=0.6. At the top 3D and bottom 2D representation of the fiber distribution is shown. The zoom shows the fine, unstructured, triangular FEM mesh.

3 MODEL DEVELOPMENT AND NUMERICAL RESULTS

The earliest and most widely applied approach in the porous media literature for predicting the permeability involves capillary models such as the one that leads to the Carman–Kozeny (CK) equation [11]. It is based on Poiseuille flow through pipe and is mainly used for 3D, homogenous, isotropic, granular porous media at moderate porosities. In the next Subsections, we present a modified CK equation which takes into account the microstructure of the fibers and is valid in a wide range of porosities.

3.1 Permeability calculations

In the CK model [11], the hydraulic diameter D_h, is expressed as a function of the measurable quantities porosity and specific surface area

$$D_h = \frac{4\varepsilon V}{S_v} = \frac{4\varepsilon}{(1-\varepsilon)a_v} = \frac{\varepsilon d}{(1-\varepsilon)}, \quad \text{with } a_v = \frac{\text{particle surface}}{\text{particle volume}} = \frac{S_v}{(1-\varepsilon)V} = \frac{4}{d} \tag{4}$$

with the total wetted surface, S_v, and the specific surface area, a_v. The above value of a_v is for circles (cylinders) – for spheres one has a_v=6/d. By applying the Poiseuille equation in terms of the hydraulic diameter as $\langle \vec{u} \rangle = -\frac{D_h^2}{32\mu}\nabla p$ and combine it with Darcy's law, Eq. (3), the normalized permeability is

$$\frac{K}{d^2} = \frac{\varepsilon^3}{\psi_{CK}\left(1-\varepsilon\right)^2} \tag{5}$$

where ψ_{CK} is the empirically measured CK factor which represents both the shape factor and the deviation of flow direction from that in a duct. It is approximated as ψ_{CK}=180 for random packed beds of spherical particles. Reported values of the CK factor for fibrous media are varying between 80 and 320 [18, 19]. The same range of ψ_{CK} has been obtained from the theoretical results of Sangani and Acrivos [9]. Since their model was primarily developed for isotropic, granular porous media of spherical particles at moderate porosity, it is necessary to re-visit and modify the model for fibrous structures composed of arrays of cylinders.
The principal limitation of the CK equation is the fact that all geometrical features of the preform are lumped into the CK factor. Even though attempts have been made to introduce microstructural features of the preform into the CK equation by suitably modifying the mean hydraulic radius, it is fair to say that, at this stage, microstructural features can be included only semi-empirically through experimental determination of ψ_{CK}. An initial attempt was made by Carman [11] who considered the effect of flow path (tortuosity) on ψ_{CK}. Writing the CK factor in terms of its components, namely the pore shape factor Φ and tortuosity L_e/L

$$\psi_{CK} = \Phi\left(\frac{L_e}{L}\right)^2 \tag{6}$$

and combine it with Eq. (5), leads to

$$\frac{K}{d^2} = \frac{\varepsilon^3}{\Phi\left(\frac{L_e}{L}\right)^2\left(1-\varepsilon\right)^2} \tag{7}$$

In the original CK equation it was assumed that the tortuosity is constant ($L_e/L = \sqrt{2}$) and Φ=90, which gives us the CK factor as ψ_{CK} =180. However, in the next Subsection we will show that the tortuosity is not constant and linearly depends on the porosity.

3.2 Measurement of the tortuosity (L_e/L)

Along with porosity, tortuosity is one of the parameters that takes into account the influence of complex microstructure on the macroscopic permeability. It is defined as the average effective streamline length scaled by the system length, L_e/L, and turns out to be a key parameter in the CK factor [11]. Experimental data for tortuosity are obtained by the measurement of the effective diffusivity [20], effective conductivity [21], acoustic wave propagation [22], and permeability [23]. The obtained values are usually model dependent and restricted to beds of spheres. Depending on factors such as packing arrangement, media homogeneity, channel shape, etc., tortuosity values in beds packed with non-uniform spheres may range from 1.7 to 4 [24]. Correlations and analytical models for tortuosity are derived from geometrical considerations on the particle scale, and usually relate the tortuosity to porosity [25]. However, they are typically restricted to spherical particles. To obtain tortuosity from our numerical simulations, we extract the average length of several streamlines (using 8 equal mass flux streamlines which divide the total mass in-flux into 9 zones, thus avoiding the centre and the edges). The tortuosity values are plotted in Fig. 2 as a function of porosity together with the best (least square) linear fit function. The tortuosity linearly depends on the porosity (with the slope of ~-0.5) at the intermediate porosity regime ($0.5<\varepsilon<0.95$). Since in the limit of $\varepsilon=1$ (no particles) one should approach the slab flow, i.e., $L_e/L=1$, the linear fit is not accurate in the very dilute regime ($\varepsilon>0.95$). By decreasing the porosity the standard deviation of the data (error bars in Fig. 2) increases which is an indication of non-homogeneity of the flow. In conclusion, we observe that (i) the tortuosity is not constant (linearly depends on porosity) and (ii) the value is smaller than $\sqrt{2}$ for uni-directional random fiber arrays for all relevant, accessible porosities.

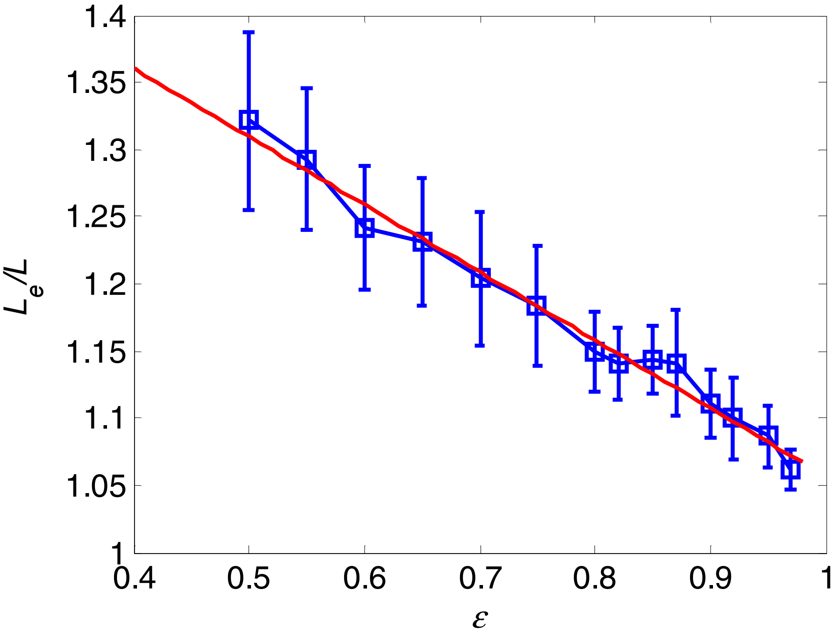

Figure 2: Variation of tortuosity L_e/L, as a function of porosity. The solid red line shows the linear best (least square) fit and the error bars indicate the standard deviations from 8 equal mass flux streamlines.

Knowing the values of L_e/L, we have fitted our numerical results into Eq. (7) to obtain new pore shape factor Φ~140 within the range $0.4<\varepsilon<0.9$. The proposed fit, based on the modified CK theory, matches the numerically calculated permeability with a maximum of 5% error in the porosity range $0.4<\varepsilon<0.9$, applicable in most composite materials. In Fig. 3 the numerical results for the normalized permeability obtained from our FEM simulations in the wide range of porosity is shown. The comparison with the available theories, namely original CK ($\psi_{CK}=180$), the modified CK ($\psi_{CK}=140(L_e/L)^2$), the results of Drummond et al. [9] based on the cell method valid at high porosity, the analytical prediction of Gebart [10] based on lubrication theory valid at low porosity, both for the hexagonal configuration, and numerical results of Chen and Papathanasiou [26] obtained from boundary element method (BEM) also displayed. For dilute systems, i.e., $\varepsilon>0.9$, the permeability data obtained from the cell approach for regular structures [9] agrees well with our numerical results. Therefore, we confirm that at high porosities, the effect of structure vanishes. At moderate porosities, i.e., 0.4<ε<0.9, the proposed modified CK equation fits better to our FEM results. However, at low porosity, i.e., ε<0.4, none of the models fit to our FEM results. This observation can be explained by looking at the microstructure (local fiber arrangement) and orientation of the narrow channels. Fig. 4 shows the distribution of the orientation of the channels θ, with respect to the flow direction, at different porosities.

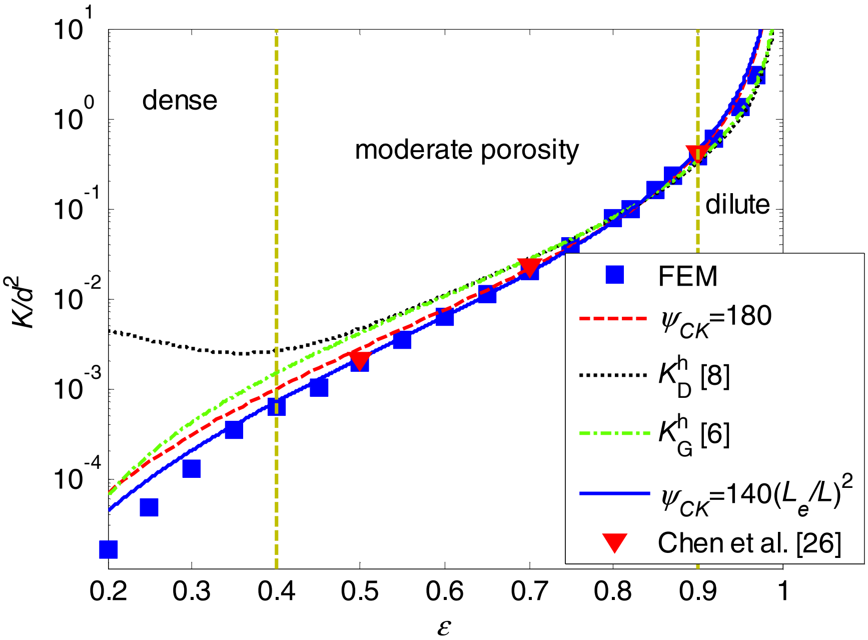

Figure 3: Comparison of normalized permeability as a function of porosity for (dis)ordered fibrous media. ψ_{CK} is the Carman-Kozeny factor. $K^h{}_G$ and $K^h{}_D$ represent the lubrication theory of Gebart [6] and the unit cell approach of Drummond and Tahir [8] both for hexagonal configurations, respectively.

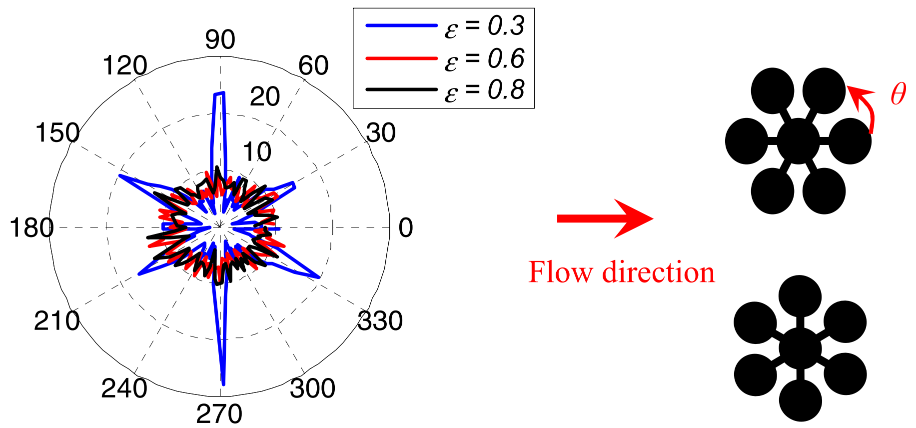

Figure 4: Plots of the probability density function (PDF) of the orientation of narrow channels at different porosities. On the right, the top shows the perfect hexagonal structure and the bottom graph, shows the schematic of the frequent hexagonal structure seen at low porosity (ε=0.3).

At high porosities we observe isotropic distributions of orientations (circular shape in red and black color in Fig. 4). However, at low porosities we see higher probabilities at 30^0, 90^0, 150^0 etc., indicating localized crystal (hexagonal) structures. In other words, in this limit we have ordered hexagonal structures which are rotated by 30^0 with respect to flow direction. The distribution of these orientations shows that they are more directed in vertical direction. Note that for ε<0.4 the effect of finite size and rigid walls/boundaries are more pronounced [17] and one might need just periodic, or much bigger systems (and/or more fibers/realizations) to obtain reliable random system and permeability data. However, the proposed modified $\psi_{CK}=140(L_e/L)^2$ is still closest to our numerical results for intermediate porosity.

4 CONCLUSIONS

A finite element based model has been employed to calculate the transverse permeability of fibrous media composed of randomly distributed long unidirectional cylinders/fibers in wide range of porosities.

It is shown that the semi-analytical Carman-Kozeny (CK) equation does not correctly capture the permeability's dependence on porosity. Therefore, an alternative relation for the permeability of the medium, taking into account the tortuosity (flow path) is developed from our numerical (FEM) results. We propose a CK coefficient, which is defined as the product of tortuosity (flow path) and pore shape factor Φ, for the intermediate porosity regimes. The modified CK equation with the new CK coefficient fits better to our numerical results in the moderate porosity range. Our results indicate that the tortuosity linearly depends on the porosity with the slope of ~-0.5 (see Fig. 2). At high porosities (ε>0.9), the random and regular configurations predict the same permeability, i.e., the effect of structure will disappear. However, at low porosities (ε<0.4), the permeability data is lower than the values obtained from lubrication theory for hexagonal configurations and the proposed fit, based on the modified CK theory. This can be explained by looking at the microstructure and local arrangement of particles/fibers (see Fig. 4).

The results obtained in this study and the modified CK relation proposed for the permeability can be utilized for composite manufacturing, e.g., resin transfer moulding processes, and also for validation of advanced models for particle-fluid interactions in a multi-scale coarse grained approach. The improvement/extension of the model for the case of different fiber shape and orientation in 3D for both creeping and inertial flow is left to be carried out in future studies.

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